

The action of neutrino ponderomotive force on Supernova dynamics

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Abstract

Collective interactions of a beam of neutrinos/antineutrinos traversing a dense magnetized plasma of electrons/positrons, protons and neutrons are studied with particular reference to the case of a Supernova. We find that the ponderomotive force exerted by neutrinos gives, contrary to expectations, a negligible contribution to the revival of the shock for a successful Supernova explosion, although new types of convection and plasma cooling processes induced by the ponderomotive force could be, in principle, relevant for the dynamics itself.

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At the end of their lives, massive stars ($M > 8M_{\odot}$) develop into Supernovae of type II and, after explosion, neutron stars are born. The mechanism of type II Supernovae is approximately well understood, although some obscure points remain [1], [2].

The iron core of the progenitor star has a mass around the Chandrasekhar limit ($\sim 1.4M_{\odot}$) so it has no stable configuration and will collapse. This is also accelerated by the negative nuclear pressure. During the collapse, the key process is the electron capture by free protons and transitions in complex nuclei, producing (electron) neutrinos that escape from the star, thus deleptonizing it. Electron capture would continue indefinitely, and the final electron fraction Y_e^f at collapse would be extremely small, were it not for the trapping of neutrinos taking place when the density in the core has reached very high values ($\rho \sim 10^{11} \div 10^{12} \text{ g/cm}^3$). After neutrinos are trapped, due to (weak neutral current mediated) elastic scattering by nuclei, they fill available phase space and their distribution can be approximately described by a Fermi-Dirac distribution with effective temperature T_{ν} and chemical potential μ_{ν} (different from those of electrons). The collapse can be stopped only when the nuclear pressure becomes positive, which happens when the nuclei touch and fuse together, forming nuclear matter. Thus, collapse continues until the central density becomes substantially greater than nuclear density ($\sim (3 \div 4) \times 10^{15} \text{ g/cm}^3$). At this point, the inner core rebounds and sends a shock wave out into the star. The shock forms not in the centre of the star, but near the surface of the homologous core (slightly outside it).

In few cases, if both the temperature at the beginning of the collapse and the mass of the iron core are low [3], the shock will go through the entire star and expel most of it, thus giving a Supernova (*prompt mechanism*). The core remains behind and will become a neutron star; the large negative gravitational energy of that neutron star provides the energy to expel the mantle and the envelope against gravitational attraction of the core. The major part of the released gravitational energy of the neutron star goes into the emission of neutrinos, which mainly occurs after the Supernova material has been set in outward motion.

Behind the shock, nuclei are dissociated into nucleons at a cost in energy of about 8.8 MeV per nucleon, and this gradually drains the energy of the shock. If the amount of material

the shock has to traverse from the homologous core before it emerges from the iron core is not small, the prompt mechanism fails, because the shock rapidly loses energy to dissociate nuclei into nucleons. The shock is therefore apt to stall at some point (typically at $r \sim 400 \text{ Km}$) and then it turns into an accretion shock in which additional infalling matter accretes to the existing core. The outward motion has then stopped and the prompt shock has failed to expel the outer part of the star. Furthermore, once the shock slows down, there is time for copious neutrino emission, which further saps its energy. However, if the shock turns into an accretion shock, neutrinos from the core can be absorbed (after about $0.5 \div 1 \text{ s}$) by material at $r \sim 100 \div 200 \text{ Km}$, and can heat this sufficiently to revive the shock, which will then expel the material from the star (*delayed mechanism*) [4].

However, in most cases, the outgoing material (ejecta) is too far away to effectively absorb the neutrinos from the core but, in numerical simulations, it is found that between ejecta and the core there is a low density region ($\rho \sim 10^7 \text{ g/cm}^3$) filled mainly by radiation (the radiation bubble) [5]. In this bubble, $e^+ e^-$ pairs can absorb neutrino energy by elastic scattering (also $\nu\bar{\nu} \rightarrow e^+e^-$ can occur), thus it continually receives new energy and continually exerts pressure on the ejecta, then driving the shock. We have to note that density increases outward near the bubble outer surface, so that dilute material in the bubble has to support and push down material in the ejecta. This causes Rayleigh-Taylor instability and convection will occur. In the delayed mechanism, convection is very fast (it is practically instantaneous compared to diffusion time scales) and reduces the energy loss by re-emission of neutrinos. Furthermore, there is an important effect. The absorption of neutrino energy takes place mainly at $r \sim 100 \div 200 \text{ Km}$; the convection brings this energy to the shock front, whether that is at $r \sim 300 \text{ Km}$ or $r \sim 4000 \text{ Km}$, and thereby continually supports the shock. This will succeed in ejecting the material outside the bubble.

These are the essential features of Supernova dynamics, although there are still some doubts especially on how the explosion mechanism can undergo effectively. In particular, it is clear that neutrinos are responsible for transporting energy from the central core to the layers surrounding it, but it seems that the collisional loss rate is marginal to produce

the required heating. Moreover, besides the key role played by neutrinos, a complete understanding of hydrodynamic instabilities and overturn before, during and after core collapse is also crucial.

In this paper we will concentrate on the possible collective interactions between neutrinos and the dense plasma in connection with the problem of the heating of the shocked envelope in a Supernova, as proposed in [6],[7]. The idea is that the material in the shock is continually run over by an intense neutrino flux, so that stimulated scattering processes can take place, in analogy with stimulated Raman or Compton scattering for laser coupling to plasma oscillations. Thus we first study the (weak interaction) ponderomotive force exerted by neutrinos on the background electrons, positrons, protons and neutrons, and then apply the results to the case of Supernova explosion.

Let us consider neutrinos with momentum \mathbf{p}_ν whose phase space distribution is $f_\nu(t, \mathbf{r}, \mathbf{p}_\nu)$ ((t, \mathbf{r}) is the 4-position), acting on the background particles $\alpha = e^\pm, p, n$ of a plasma with momentum \mathbf{p}_α , distributed according to $f_\alpha(t, \mathbf{r}, \mathbf{p}_\alpha)$, through an effective potential

$$\mathcal{V}_{eff}^\alpha = 2 \int \frac{d\mathbf{p}_\alpha}{(2\pi)^3} V_{eff}^\alpha(t, \mathbf{r}, \mathbf{p}_\alpha) \quad (1)$$

(the factor 2 is the statistical weight for the fermions $\alpha = e^\pm, p, n$). The force exerted by neutrinos over (a single) background particle $\alpha = e^\pm, p, n$ can then be written in the form [8]

$$\begin{aligned} \mathcal{F}_\nu^\alpha &= \frac{1}{n_\alpha} 2 \int \frac{d\mathbf{p}_\nu}{(2\pi)^3} \int \frac{d\mathbf{p}_\alpha}{(2\pi)^3} \left(f_\nu(t, \mathbf{r}, \mathbf{p}_\nu) \nabla V_{eff}^\alpha(t, \mathbf{r}, \mathbf{p}_\alpha) + \right. \\ &\quad \left. - \nabla \left(f_\nu(t, \mathbf{r}, \mathbf{p}_\nu) \left(\frac{\partial V_{eff}^\alpha}{\partial f_\alpha} \right)_T f_\alpha(t, \mathbf{r}, \mathbf{p}_\alpha) \right) \right) \end{aligned} \quad (2)$$

where the particle number densities are

$$n_\alpha(t, \mathbf{r}) = 2 \int \frac{d\mathbf{p}_\alpha}{(2\pi)^3} f_\alpha(t, \mathbf{r}, \mathbf{p}_\alpha) \quad (3)$$

$$n_\nu(t, \mathbf{r}) = \int \frac{d\mathbf{p}_\nu}{(2\pi)^3} f_\nu(t, \mathbf{r}, \mathbf{p}_\nu) \quad (4)$$

(neutrinos are completely polarized particles). We now have to calculate the effective potential for $\nu - \alpha$ coherent interactions. Let us first note that in Supernovae large magnetic

fields \mathbf{B} are, in general, present [9] so that neutrino interaction with the background is modified with respect to the case in which the medium is non magnetized. However, since the calculation of the effective potential proceeds from that of neutrino self-energy in the considered medium [10], [11], [12], in general one can write

$$V_{eff}^{tot} = V_{eff}^{\mathbf{B}=\mathbf{0}} + V_{eff}^{\mathbf{B}\neq\mathbf{0}} \quad (5)$$

where the first term refers to the $\mathbf{B} = \mathbf{0}$ case while the second one explicitly involves the presence of the magnetic field.

Let us first examine the contribution independent on \mathbf{B} ². Two kind of Feynman diagrams contribute to neutrino self-energy [10]: one involves charged current interaction between ν_e and electrons while in the others we are concerned with neutral current interactions between each neutrino flavour and electrons, protons and neutrons. For neutrino-electron interactions we have

$$V_{eff}^{e-} = \sqrt{2} G_F g_V f_{e-}(t, \mathbf{r}, \mathbf{p}_{e-}) \quad (6)$$

where G_F is the Fermi coupling constant and

$$g_V = g_V^{CC} + g_V^{NC} \quad \text{for } \nu_e \quad (7)$$

$$g_V = g_V^{NC} \quad \text{for } \nu_\mu, \nu_\tau \quad (8)$$

$$g_V^{CC} = 1 \quad , \quad g_V^{NC} = 2 \sin^2 \theta_W - \frac{1}{2} \quad (9)$$

with θ_W the Weinberg angle of the electro-weak standard model. The effective potentials for neutrino-positron and neutrino-proton are instead given by

$$V_{eff}^{e+} = -\sqrt{2} G_F g_V f_{e+}(t, \mathbf{r}, \mathbf{p}_{e+}) \quad (10)$$

$$V_{eff}^p = -\sqrt{2} G_F g_V^{NC} f_p(t, \mathbf{r}, \mathbf{p}_p) \quad . \quad (11)$$

Finally, neutrino-neutron interactions are described by

$$V_{eff}^n = -\frac{G_F}{\sqrt{2}} g_V f_n(t, \mathbf{r}, \mathbf{p}_n) \quad . \quad (12)$$

²In what follows we consider only isotropic media or, more in general, anisotropic ones with equal fluxes of electrons moving in opposite directions (see [12]).

Note also that the effective potential for antineutrino interactions is opposite to that for neutrino. Then, from Eq.(2), we obtain the $\mathbf{B} = \mathbf{0}$ contribution to the ponderomotive force experienced by e^\pm , p , n due to a beam of neutrinos/antineutrinos:

$$\mathcal{F}_\nu^{e^-} = -\mathcal{F}_\nu^{e^+} = -\sqrt{2} G_F \left(\nabla (n_{\nu_e} - n_{\bar{\nu}_e}) + g_V^{NC} \nabla (n_\nu - n_{\bar{\nu}}) \right) \quad (13)$$

$$\mathcal{F}_\nu^p = \sqrt{2} G_F g_V^{NC} \nabla (n_\nu - n_{\bar{\nu}}) \quad (14)$$

$$\mathcal{F}_\nu^n = \frac{G_F}{\sqrt{2}} \nabla (n_\nu - n_{\bar{\nu}}) \quad (15)$$

$$(n_\nu = n_{\nu_e} + n_{\nu_\mu} + n_{\nu_\tau}).$$

Let us now consider the contribution induced by the presence of the magnetic field (the second term in (5)), which we suppose to lie in the positive z direction. For neutrino-electron interactions, by indicating with λ_z the polarization of the electrons in the plasma, we have [12]

$$V_{eff}^{e^-} = -\sqrt{2} G_F g_A \cos \alpha_B \lambda_z f_e(p_{ez}, n, \lambda_z) \quad (16)$$

where $\cos \alpha_B = \hat{\mathbf{p}}_\nu \cdot \hat{\mathbf{B}}$ and

$$g_A = g_A^{CC} + g_A^{NC} \quad \text{for } \nu_e \quad (17)$$

$$g_A = g_A^{NC} \quad \text{for } \nu_\mu, \nu_\tau \quad (18)$$

$$g_A^{CC} = -1 \quad g_A^{NC} = \frac{1}{2} \quad . \quad (19)$$

In (16), $f_e(p_{ez}, n, \lambda_z)$ is the electron distribution function (for simplicity we have suppressed the spatial dependence) which for a magnetized medium takes the form

$$f_e(p_{ez}, n, \lambda_z) = \left(1 + \exp \left\{ \frac{\epsilon(p_{ez}, n, \lambda_z) - \mu}{T} \right\} \right)^{-1} \quad (20)$$

where $\epsilon(p_z, n, \lambda_z)$ are the quantized Landau energy levels given by ($e > 0$)

$$\epsilon(p_{ez}, n, \lambda_z) = \sqrt{p_{ez}^2 + m_e^2 + e B (2n + 1 + \lambda_z)} \quad (21)$$

with $n = 0, 1, 2, \dots$, $\lambda_z = \pm 1$. For neutrino-positron interactions the same form in (16) holds, but with the replacements $V_{eff} \rightarrow -V_{eff}$ and $\mu \rightarrow -\mu$. Instead for neutrino-nucleon interactions the effective potential is still proportional to the polarization of the considered background particle, but now, for the case of interest in a Supernova, in which

nucleons are strongly (or not weakly) non degenerate, this will be given by the proton $\mu_p = 2.79 e/2M_N$ or neutron $\mu_n = -1.91 e/2M_N$ (M_N is the nucleon mass) magnetic moments. We therefore have [12]

$$\mathcal{V}_{eff}^p = -\frac{G_F}{\sqrt{2}} \frac{C_A^N}{C_V^N} n_p \frac{\mu_p B}{T} \cos \alpha_B \quad (22)$$

$$\mathcal{V}_{eff}^n = \frac{G_F}{\sqrt{2}} \frac{C_A^N}{C_V^N} n_n \frac{\mu_n B}{T} \cos \alpha_B \quad (23)$$

where $C_A^N/C_V^N \simeq 1.26$ is the axial to vector nucleon form factor.

Let us now pass to calculate the ponderomotive force induced by the effective potential in (16); first observe that, since the energy levels are quantized, we have

$$2 \int \frac{d\mathbf{p}_e}{(2\pi)^3} \longrightarrow \sum_{n=0}^{\infty} \sum_{\lambda_z} 2\pi eB \int_{-\infty}^{\infty} \frac{dp_{ez}}{(2\pi)^3} \quad . \quad (24)$$

Then from Eq. (2) we get

$$\mathcal{F}_\nu^{e-} = \sqrt{2} G_F g_A \sum_{n=0}^{\infty} \sum_{\lambda_z} 2\pi \frac{eB}{(2\pi)^2} \int_{-\infty}^{\infty} dp_{ez} \lambda_z f_e(p_{ez}, n, \lambda_z) \int \frac{d\mathbf{p}_\nu}{(2\pi)^3} \cos \alpha_B \nabla f_\nu \quad . \quad (25)$$

Since f_ν does not depend on neutrino angles,

$$\int \frac{d\mathbf{p}_\nu}{(2\pi)^3} \cos \alpha_B \nabla f_\nu = \frac{1}{(2\pi)^3} \int p_\nu^2 dp_\nu \nabla f_\nu \int_{-1}^1 d\cos \alpha_B \cos \alpha_B = 0 \quad (26)$$

and so

$$\mathcal{F}_\nu^{e-} = 0 \quad . \quad (27)$$

In a similar fashion, we can also prove that

$$\mathcal{F}_\nu^{e+} = \mathcal{F}_\nu^p = \mathcal{F}_\nu^n = 0 \quad . \quad (28)$$

We then have the important result that the magnetic field dependent term in the effective potential (5) does not contribute to the ponderomotive force of neutrinos by virtue of the dependence of $V_{eff}^{\mathbf{B}} \neq 0$ on $\cos \alpha_B = \hat{\mathbf{p}}_\nu \cdot \hat{\mathbf{B}}$, and thus Eqs. (13)-(15) are general results.

Some considerations are now in order. Due to weak couplings, we have that, as regards nucleons,

$$|\mathcal{F}_\nu^p| \approx 8\% |\mathcal{F}_\nu^n| \quad (29)$$

so the neutrino force on neutrons is more efficient than that on protons; secondly, for a given neutrino net number, the neutral current contribution to \mathcal{F}_ν^e is about 4 % of that

of charged current one.

Considering the typical situation present in a Supernova, we can also predict the sign of the ponderomotive force exerted by neutrinos. For ν_μ, ν_τ the neutrino-nucleon elastic scattering cross section is (slightly) higher than the one for antineutrino-nucleon [13], so antineutrinos escape more easily from the star, leaving the Supernova more rich in neutrinos:

$$n_{\nu_{\mu,\tau}} - n_{\bar{\nu}_{\mu,\tau}} > 0 \quad . \quad (30)$$

The same approximately holds for the thermally produced $\nu_e, \bar{\nu}_e$; moreover there are also ν_e from the deleptonization of the Supernova core, and then

$$n_{\nu_e} - n_{\bar{\nu}_e} > 0 \quad . \quad (31)$$

Then, the net neutrino number density is a decreasing function of the distance r from the centre of the star, so that its gradient is negative:

$$\nabla (n_{\nu_l} - n_{\bar{\nu}_l}) < 0 \quad . \quad (32)$$

Summing up, we found that neutrino ponderomotive force is negative (i.e. attractive towards the centre ($\propto -\mathbf{r}$), assuming spherical symmetry) on neutrons and positrons, while is positive (i.e. repulsive from the centre) for protons and electrons.

Such a ponderomotive force exerted by neutrinos can play, in principle, an important role in Supernova dynamics. The major effect would be related to the revival of the shock by means of the energy deposited in the background material behind the shock itself. In the standard delayed mechanism, if σ is the averaged cross section for neutrino-electron and neutrino-nucleon scattering,

$$\sigma(\nu e \rightarrow \nu e) \simeq k G_F^2 E_\nu T \quad (33)$$

$$\sigma(\nu_e n \rightarrow e^- p) \simeq \omega_1 G_F^2 E_\nu^2 \quad (34)$$

$$\sigma(\bar{\nu}_e p \rightarrow e^+ n) \simeq \omega_2 G_F^2 E_\nu^2 \quad (35)$$

(where k is a given constant depending on the particular channel, while ω_1, ω_2 are (smooth) functions of neutrino energy E_ν ; see [14] for more details), the energy gain of a given

particle in the background at a distance R is [1]

$$\frac{dE}{dt} = \frac{\mathcal{L} \sigma}{4\pi R^2} \quad (36)$$

\mathcal{L} being the luminosity in neutrinos or antineutrinos. This relation is not, in general, complete since the material absorbing the neutrinos can also emit neutrinos spontaneously; the total energy change is then given by (36) times a correction factor estimated as [1]

$$1 - \left(\frac{2R}{R_\nu}\right)^2 \left(\frac{T}{T_\nu}\right)^6 \quad (37)$$

where R_ν , T_ν are the radius and temperature of the neutrinosphere (while T is the material temperature). For typical numbers,

$$\mathcal{L} \approx 5 \times 10^{52} \text{ ergs/s}$$

$$\langle E_\nu^2 \rangle \approx 6T_\nu^2 \approx 100 \text{ MeV} \quad (38)$$

$$R \approx 150 \text{ Km} \quad (39)$$

and assuming that all nucleons are free behind the shock, the absorption average energy gain per nucleon is (from (36))

$$\frac{dE}{dt} \simeq 50 \text{ Mev/s} \quad (40)$$

while for neutrino-electron scattering the rate is about 1/2 of this, due to the different cross section.

However, there are also collective interactions of neutrinos on the background plasma described by the ponderomotive force in (13)-(15). The energy change induced on e^\pm, p, n in the material by this force is obtained from

$$\frac{dE_\alpha}{dt} = \mathcal{F}_\nu^\alpha \cdot \mathbf{v}_\alpha \quad (41)$$

where \mathbf{v}_α is the velocity in the medium of the considered particle. By comparison of (41) and (36) we immediately see that, as expected [7]³, collective effects are in general more relevant than incoherent ones since, while the rate in (36) is proportional to G_F^2 , the expression in (41) is of order G_F and then the mechanism more efficient. However, for the

³In [7] there is an overestimation of the momentum transfer from the ponderomotive force to the background, leading to exceedingly large values for the neutrino force.

present case, there are also two important suppression factors to be taken into account, namely the dependence of \mathcal{F}_ν on the difference between neutrinos and antineutrinos number densities (which would be almost equal for $\nu, \bar{\nu}$ produced thermally by $e^+e^- \rightarrow \nu\bar{\nu}$) and the dependence of dE/dt on the average angle between the ponderomotive force and the velocity of the background particle. Let us first calculate the maximum energy gain implied by (41); to this end we have first to estimate the net neutrino number in a Supernova. We adopt a very simple model in which spherical symmetry is assumed: neutrinos are emitted, with a Fermi-Dirac distribution of temperature T_ν , from the neutrinosphere (practically coincident with the core surface) located at R_ν and escape freely outward from that. No neutrino source is considered outside the neutrinosphere, so that the neutrino flux decrease with distance only for a dilution spatial factor,

$$n_{\nu_l} - n_{\bar{\nu}_l} = (n_{\nu_l} - n_{\bar{\nu}_l})_0 \left(\frac{R_\nu}{r} \right)^2 \quad (42)$$

(for $r > R_\nu$) where $(n_{\nu_l} - n_{\bar{\nu}_l})_0$ is the neutrino net number density at neutrinosphere, which we now pass to evaluate. Let us first concern on non electron neutrinos, that are produced entirely by thermal processes. The difference between neutrino and antineutrino distribution arises in this case only from the (slightly) different neutrino-nucleon elastic scattering cross section, and vanishes in the infinite nucleon mass approximation. The net number is then proportional to T/M_N (M_N is the nucleon mass) and is calculated in [13]; at the neutrinosphere we have

$$(n_{\nu_x} - n_{\bar{\nu}_x})_0 \simeq \frac{\pi^2}{36} \delta \left(\frac{T_{\nu_x}}{M_N} \right) T_{\nu_x}^3 \quad (43)$$

($x = \mu, \tau$) where δ is a parameter depending on nucleon weak couplings, equal to 3.32 for neutrons and 2.71 for protons.

The difference in ν_e and $\bar{\nu}_e$ distributions induced by the different neutral current scattering is the same as in (43); however, for electron neutrinos and antineutrinos there is a more important contribution coming from charged current (elastic and inelastic) scattering. This induces a (not large) effective degeneracy parameter $\eta_\nu = \mu_\nu/T_\nu$, and the net electron neutrino number is approximately proportional to this parameter:

$$(n_{\nu_e} - n_{\bar{\nu}_e})_0 \simeq \frac{1}{6} \eta_\nu T_\nu^3 \quad . \quad (44)$$

Following [1], η_ν can be estimated from the fact that in a Supernova the ratio of the (average) energies in ν_e and $\nu_e + \bar{\nu}_e$, due to the deleptonization electron neutrinos from the core, is approximately equal to 5/8. From this we have

$$\eta_\nu \simeq 0.29 \quad (45)$$

(but this result depends somewhat on T_ν and on the average neutrino chemical potential in the core).

From the above arguments we are now able to evaluate quantitatively the ponderomotive force exerted by neutrinos; we get ⁴

$$\begin{aligned} \mathcal{F}_\nu^{e^-} &= -\mathcal{F}_\nu^{e^+} \simeq \\ &\simeq 2.94 \times 10^{-6} \left\{ \left(\frac{\eta_{\nu_e}}{0.29} \right) \left(\frac{T_{\nu_e}}{4 \text{ MeV}} \right)^3 - 3.2 \times 10^{-2} \left(\frac{\delta}{3.32} \right) \left(\frac{T_{\nu_x}}{6 \text{ MeV}} \right)^4 \right\} \cdot \\ &\cdot \left(\frac{10 \text{ Km}}{R_\nu} \right) \left(\frac{R_\nu}{r} \right)^3 \hat{\mathbf{r}} \frac{\text{MeV}}{c} s^{-1} \end{aligned} \quad (46)$$

$$\begin{aligned} \mathcal{F}_\nu^p &\simeq 1.2 \times 10^{-7} \left\{ \left(\frac{\eta_{\nu_e}}{0.29} \right) \left(\frac{T_{\nu_e}}{4 \text{ MeV}} \right)^3 + 0.79 \left(\frac{\delta}{3.32} \right) \left(\frac{T_{\nu_x}}{6 \text{ MeV}} \right)^4 \right\} \cdot \\ &\cdot \left(\frac{10 \text{ Km}}{R_\nu} \right) \left(\frac{R_\nu}{r} \right)^3 \hat{\mathbf{r}} \frac{\text{MeV}}{c} s^{-1} \end{aligned} \quad (47)$$

$$\begin{aligned} \mathcal{F}_\nu^n &\simeq -1.5 \times 10^{-6} \left\{ \left(\frac{\eta_{\nu_e}}{0.29} \right) \left(\frac{T_{\nu_e}}{4 \text{ MeV}} \right)^3 + 0.79 \left(\frac{\delta}{3.32} \right) \left(\frac{T_{\nu_x}}{6 \text{ MeV}} \right)^4 \right\} \cdot \\ &\cdot \left(\frac{10 \text{ Km}}{R_\nu} \right) \left(\frac{R_\nu}{r} \right)^3 \hat{\mathbf{r}} \frac{\text{MeV}}{c} s^{-1} \end{aligned} \quad (48)$$

Note that, for a given distance, the force acting on electrons is about a factor 13 greater than the one acting on protons, while electrons and neutrons experience about the same force. Observe also that while the contribution of ν_μ, ν_τ is subdominant with respect to that of ν_e in $\mathcal{F}_\nu^{e^-}$, each flavour of neutrinos contribute with about the same strength in $\mathcal{F}_\nu^p, \mathcal{F}_\nu^n$.

We can now estimate the maximum energy gain of the Supernova material behind the shock front due to the considered collective effects by assuming that electrons are relativistic while the mean nucleon velocity is given by $\sim \sqrt{3T/M_N} \approx 0.06$. For $R = 150 \text{ Km}$

⁴For illustrative purposes, we take the value 3.32 for δ and consider e -neutrinosphere and x -neutrinosphere as located at about the same radius R_ν , although with different temperature. This last approximation is justified from the fact that we will concern with effects induced by the ponderomotive force taking place not very near the neutrinospheres. Obviously, both these assumptions have to be relaxed in a detailed Supernova numerical simulation.

we clearly find that the electron energy gain is about a factor 3×10^{-11} less than that of the standard results, and the electron heating is the major effect for collective interactions, contrary to the standard picture in which the nucleon heating (by inelastic scattering) is the most remarkable one. Moreover, in the proposed scenario, another suppression factor comes from the dependence on the angle between electron (or nucleon) motion and the ponderomotive force (see Eq. (41)). In fact, the previous estimate of the heating rate assumes that nearly all the electrons (or nucleons) are polarized; then, although the magnetic field contribution to \mathcal{F}_ν is unexisting (as established here), it would be nevertheless important for reaching the maximum effect ⁵, which is, however, completely subdominant with respect to the standard heating mechanism, the heating rate being some $eV s^{-1}$ for $r = R_\nu$ and decreasing as r^{-3} .

Nonetheless, the action of the ponderomotive force due to neutrinos is not, in general, restricted to heat the material behind the shock (at least in a direct way). It could give a non negligible contribution to the theory of convection and instabilities during a Supernova explosion. In fact we remind, from (46)-(48), that while $\mathcal{F}_\nu^{e^-}$ and \mathcal{F}_ν^p are positive, $\mathcal{F}_\nu^{e^+}$ and \mathcal{F}_ν^n are negative. This means that e^- and p are pushed in the outgoing radial direction, while e^+ and n in the ingoing one, and this could be a relevant source of convection, as pointed out in [8], especially near the radiation bubble, where Rayleigh-Taylor instabilities already act.

In conclusion, we have studied collective interactions of a beam of neutrinos/antineutrinos traversing a dense plasma of e^\pm, p, n and applied the results to the case of a Supernova. We have found that the ponderomotive force exerted by neutrinos on the material behind the shock cannot substantially heat that and gives a negligible contribution to the revival of the shock for a successful Supernova explosion. In particular, the magnetic field present in the considered plasma does not contribute to the expression of the ponderomotive force, although it is relevant for the polarization effects induced by it in the medium.

The collective interactions studied here can, nevertheless, play a relevant role in dense

⁵In [12] the mean electron polarization in a Supernova for the Wilson & Mayle model has been calculated. The authors find that for $r \simeq 100 \div 200 \text{ Km}$ it can be very high (near the unit value) allowing the magnetic field in this region to be as high as $\sim 10^{16} \text{ Gauss}$.

stellar plasmas [15] where they can provide an additional plasma cooling process through neutrino Landau damping of electron plasma waves, thus influencing the evolution of massive stars.

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